GLOBAL EDITION

Essential University Physics

Volume 2

THIRD EDITION

Richard Wolfson

ALWAYS LEARNING

PEARSON

ESSENTIAL UNIVERSITY PHYSICS

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Middlebury College

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PhET Simulations

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Chapter	PhET	Page
1	Estimation	26
2	The Moving Man	34
3	Vector Addition	51
3	Projectile Motion	57
3	Ladybug Motion 2D	61
3	Motion in 2D	61
5	The Ramp	91
5	Forces in 1 Dimension	98
5	Friction	98
6	The Ramp	110, 114
7	Calculus Grapher	138
7	Energy Skate Park	138
8	Gravity Force Lab	149
8	My Solar System	150
8	Gravity and Orbits	150
9	Collisions (Introduction)	176
9	Collisions (Advanced)	177
10	Ladybug Revolution	187
10	Torque (Torque)	191
10	Torque (Moment of Inertia)	191
11	Simplified MRI	214
13	Masses and Springs	242
13	Pendulum Lab	247
14	Wave on a String	265, 273
14	Sound	269
14	Fourier: Making Waves	270
14	Wave Interference	271
15	Balloons and Buoyancy	287
16	Blackbody Spectrum	311
16	The Greenhouse Effect	314
17	Gas Properties	321
17	States of Matter	325
19	Reversible Reactions	362
20	Balloons and Static Electricity	436
20	Charges and Fields	439, 442
20	Electricity Field Hockey	446
22	Calculus Grapher	489
22	Charges and Fields	489
24	Conductivity	515
24	Semiconductors	517
24	Resistance in a Wire	519
24	Ohm's Law	519
25	Battery Resistor Circuit	530
25	Circuit Construction Kit (DC Only)	531, 536
25	Signal Circuit	531
26	Magnet and Compass	550
26	Magnets and Electromagnets	563
27	Farady's Law	578
27	Farady's Electromagentic Lab	582
27	Generator	586
28	Circuit Construction Kit (AC + DC)	606, 610, 613
29	Radio Waves and Electromagnetic Fields	636
29	Radiating Charge	636
29	Optical Tweezers and Applications	639
30	Bending Light (Intro)	649
30	Bending Light (Prism Break)	652
31	Geometric Optics	665
32	Wave Interference: Light	681
34	Blackbody Spectrum	728
34	Photoelectric Effect	731
34	Neon Lights & Other Discharge Lamps	734
34	Quantum Wave Interference	739
35	Quantum Bound State: One Well	751, 755
35	Quantum Tunneling and Wave Packets	757
36	Quantum Bound State: One Well: 3D Coulomb	765
36	Build an Atom	773
37	Quantum Bound States: Two Wells (Molecular Bonding)	783
37	Band Structure	789
37	Semiconductors	792
38	Rutherford Scattering	801
38	Simplified MRI	804
38	Radioactive Dating Game	806
38	Alpha Decay	809
38	Beta Decay	809
38	Nuclear Fission: One Nucleus	814
38	Nuclear Fission: Chain Reaction	815
38	Nuclear Fission: Nuclear Reactor	816

Video Tutor Demonstrations

Video tutor demonstrations can be accessed by scanning the QR codes in the textbook using a smartphone. They are also available in the Study Area and Instructor's Resource Area on MasteringPhysics and in the eText.

Chapter	Video Tutor Demonstration	Page
2	Balls Take High and Low Tracks	34
3	Dropped and Thrown Balls	56
3	Ball Fired from Cart on Incline	57
3	Ball Fired Upward from Accelerating Cart	57
3	Range of a Gun at Two Firing Angles	61
4	Cart with Fan and Sail	71
4	Ball Leaves Circular Track	71
4	Suspended Balls: Which String Breaks?	72
4	Weighing a Hovering Magnet	81
5	Tension in String between Hanging Weights	94
7	Chin Basher?	136
9	Balancing a Meter Stick	164
9	Water Rocket	171
9	Happy/Sad Pendulums	172
10	Canned Food Race	200
11	Spinning Person Drops Weights	213
11	Off-Center Collision	213
12	Walking the Plank	225
13	Vibrating Rods	254
14	Out-of-Phase Speakers	269
15	Pressure in Water and Alcohol	286
15	Water Level in Pascal's Vases	286
15	Weighing Weights in Water	288
15	Air Jet Blows between Bowling Balls	292
16	Heating Water and Aluminum	306
16	Water Balloon Held over Candle Flame	306
16	Candle Chimneys	311
20	Charged Rod and Aluminum Can	449
21	Electroscope in Conducting Shell	471
22	Charged Conductor with Teardrop Shape	491
23	Discharge Speed for Series and Parallel Capacitors	503
24	Resistance in Copper and Nichrome	521
25	Bulbs Connected in Series and in Parallel	534
26	Magnet and Electron Beam	550
26	Current-Carrying Wire in Magnetic Field	555
27	Eddy Currents in Different Metals	587
29	Parallel-Wire Polarizer for Microwaves	633
29	Point of Equal Brightness between Two Light Sources	638
31	Partially Covering a Lens	667
36	Illuminating Sodium Vapor with Sodium and Mercury Lamps	776

Brief Contents

Chapter 1 Doing Physics 19

PART ONE

Mechanics 32

Chapter 2 Motion in a Straight Line 33

Chapter 3 Motion in Two and Three Dimensions 50

Chapter 4 Force and Motion 69

Chapter 5 Using Newton's Laws 89

Chapter 6 Energy, Work, and Power 108

Chapter 7 Conservation of Energy 127

Chapter 8 Gravity 147

Chapter 9 Systems of Particles 162

Chapter 10 Rotational Motion 186

Chapter 11 Rotational Vectors and Angular Momentum 207

Chapter 12 Static Equilibrium 222

PART TWO

Oscillations, Waves, and Fluids 239

Chapter 13 Oscillatory Motion 240

Chapter 14 Wave Motion 261

Chapter 15 Fluid Motion 283

PART THREE

Thermodynamics 302

Chapter 16 Temperature and Heat 303

Chapter 17 The Thermal Behavior of Matter 321

Chapter 18 Heat, Work, and the First Law of Thermodynamics 335

Chapter 19 The Second Law of Thermodynamics 352

PART FOUR

Electromagnetism 434

Chapter 20 Electric Charge, Force, and Field 435

Chapter 21 Gauss's Law 455

Chapter 22 Electric Potential 479

Chapter 23 Electrostatic Energy and Capacitors 498

Chapter 24 Electric Current 512

Chapter 25 Electric Circuits 529

Chapter 26 Magnetism: Force and Field 549

Chapter 27 Electromagnetic Induction 577

Chapter 28 Alternating-Current Circuits 605

Chapter 29 Maxwell's Equations and Electromagnetic Waves 623

PART FIVE

Optics 645

Chapter 30 Reflection and Refraction 646

Chapter 31 Images and Optical Instruments 659

Chapter 32 Interference and Diffraction 679

PART SIX

Modern Physics 701

Chapter 33 Relativity 702

Chapter 34 Particles and Waves 727

Chapter 35 Quantum Mechanics 747

Chapter 36 Atomic Physics 764

Chapter 37 Molecules and Solids 782

Chapter 38 Nuclear Physics 800

Chapter 39 From Quarks to the Cosmos 827

APPENDICES

Appendix A Mathematics 847

Appendix B The International System of Units (SI) 855

Appendix C Conversion Factors 857

Appendix D The Elements 859

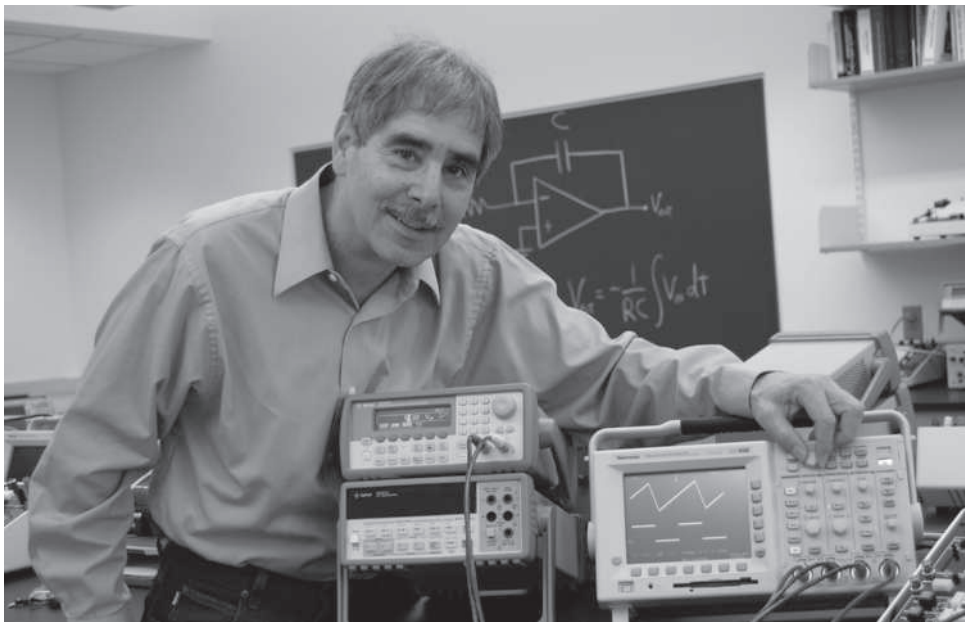
Appendix E Astrophysical Data 862

Answers to Odd-Numbered Problems 863

Credits 872

Index 875

About the Author



Richard Wolfson

Richard Wolfson is the Benjamin F. Wissler Professor of Physics at Middlebury College, where he has taught since 1976. He did undergraduate work at MIT and Swarthmore College, and he holds an M.S. degree from the University of Michigan and Ph.D. from Dartmouth. His ongoing research on the Sun's corona and climate change has taken him to sabbaticals at the National Center for Atmospheric Research in Boulder, Colorado; St. Andrews University in Scotland; and Stanford University.

Rich is a committed and passionate teacher. This is reflected in his many publications for students and the general public, including the video series *Einstein's Relativity and the Quantum Revolution: Modern Physics for Nonscientists* (The Teaching Company, 1999), *Physics in Your Life* (The Teaching Company, 2004), *Physics and Our Universe: How It All Works* (The Teaching Company, 2011), and *Understanding Modern Electronics* (The Teaching Company, 2014); books *Nuclear Choices: A Citizen's Guide to Nuclear Technology* (MIT Press, 1993), *Simply Einstein: Relativity Demystified* (W. W. Norton, 2003), and *Energy, Environment, and Climate* (W. W. Norton, 2012); and articles for *Scientific American* and the *World Book Encyclopedia.*

Outside of his research and teaching, Rich enjoys hiking, canoeing, gardening, cooking, and watercolor painting.

Preface to the Instructor

Introductory physics texts have grown ever larger, more massive, more encyclopedic, more colorful, and more expensive. *Essential University Physics* bucks that trend—without compromising coverage, pedagogy, or quality. The text benefits from the author's three decades of teaching introductory physics, seeing firsthand the difficulties and misconceptions that students face as well as the "Got It!" moments when big ideas become clear. It also builds on the author's honing multiple editions of a previous calculus-based textbook and on feedback from hundreds of instructors and students.

Goals of This Book

Physics is the fundamental science, at once fascinating, challenging, and subtle—and yet simple in a way that reflects the few basic principles that govern the physical universe. My goal is to bring this sense of physics alive for students in a range of academic disciplines who need a solid calculus-based physics course—whether they're engineers, physics majors, premeds, biologists, chemists, geologists, mathematicians, computer scientists, or other majors. My own courses are populated by just such a variety of students, and among my greatest joys as a teacher is having students who took a course only because it was required say afterward that they really enjoyed their exposure to the ideas of physics. More specifically, my goals include:

- Helping students build the analytical and quantitative skills and confidence needed to apply physics in problem solving for science and engineering.
- Addressing key misconceptions and helping students build a stronger conceptual understanding.
- Helping students see the relevance and excitement of the physics they're studying with contemporary applications in science, technology, and everyday life.
- Helping students develop an appreciation of the physical universe at its most fundamental level.
- Engaging students with an informal, conversational writing style that balances precision with approachability.

New to the Third Edition

The overall theme for this third-edition revision is to present a more unified view of physics, emphasizing "big ideas" and the connections among different topics covered throughout the book. We've also updated material and features based on feedback from instructors, students, and reviewers. A modest growth, averaging about one page per chapter, allows for expanded coverage of topics where additional elaboration seemed warranted. Several chapters have had major rewrites of key physics topics. We've also made a number of additions and modifications aimed at improving students' understanding, increasing relevancy, and offering expanded problem-solving opportunities.

- Chapter opening pages have been redesigned to include explicit connections, both textual and graphic, with preceding and subsequent chapters.
- The presentation of **energy and work** in Chapters 6 and 7 has been extensively rewritten with a clearer invocation of **systems concepts**. Internal energy is introduced much earlier in the book, and potential energy is carefully presented as a property not of objects but of systems. Two new sections in Chapter 7 emphasize the universality of energy conservation, including the role of internal energy in systems subject to dissipative forces. Forward references tie this material to the chapters on thermodynamics, electromagnetism, and relativity. The updated treatment of energy also allows the text to make a closer connection between the conservation laws for energy and momentum.

- The presentation of **magnetic flux and Faraday's law** in Chapter 27 has been recast so as to distinguish motional emf from emfs induced by changing magnetic fields—including Einstein's observation about induction, which is presented as a forward-looking connection to Chapter 33.
- There is more emphasis on calculus in earlier chapters, allowing instructors who wish to do so to use calculus approaches to topics that are usually introduced algebraically. We've also added more calculus-based problems. However, we continue to emphasize the standard approach in the main text for those who teach the course with a calculus corequisite or otherwise want to go slowly with more challenging math.
- A host of **new applications** connects the physics concepts that students are learning with contemporary technological and biomedical innovations, as well as recent scientific discoveries. A sample of new applications includes Inertial Guidance Systems, Vehicle Stability Control, Climate Modeling, Electrophoresis, MEMS (Microelectromechanical Systems), The Taser, Uninterruptible Power Supplies, Geomagnetic Storms, PET Scans, Noise-Cancelling Headphones, Femtosecond Chemistry, Windows on the Universe, and many more.
- Additional **worked examples** have been added in areas where students show the need for more practice in problem solving. Many of these are not just artificial textbook problems but are based on contemporary science and technology, such as the Mars *Curiosity* rover landing, the Fukushima accident, and the Chelyabinsk meteor. Following user requests, we've added an example of a collision in the center-of-mass reference frame.
- New GOT IT? boxes, now in nearly every section of every chapter, provide quick checks on students' conceptual understanding. Many of the GOT IT? questions have been formatted as Clicker questions, available in the Instructor's Resource Area in Mastering.
- End-of chapter problem sets have been extensively revised:
  - Each EOC problem set has at least 10 percent new or substantially revised problems.
  - More "For Thought and Discussion Questions" have been added.
  - Nearly every chapter has more intermediate-level problems.
  - More calculus-based problems have been added.
  - Every chapter now has at least one data problem, designed to help students develop strong quantitative reasoning skills. These problems present a data table and require students to determine appropriate functions of the data to plot in order to achieve a linear relationship and from that to find values of physical quantities involved in the experiment from which the data were taken.
  - New tags have been added to label appropriate problems. These include CH (challenge), ENV (environmental), and DATA, and they join the previous BIO and COMP (computer) problem tags.
- QR codes in margins allow students to use smartphones or other devices for immediate access to video tutor demonstrations that illustrate selected concepts while challenging students to interact with the video by predicting outcomes of simple experiments.
- References to PhET simulations appear in the margins where appropriate.
- As with earlier revisions, we've incorporated new research results, new applications of physics principles, and findings from physics education research.

Pedagogical Innovations

This book is *concise*, but it's also *progressive* in its embrace of proven techniques from physics education research and *strategic* in its approach to learning physics. Chapter 1 introduces the IDEA framework for problem solving, and every one of the book's subsequent **worked examples** employs this framework. IDEA—an acronym for Identify, Develop, Evaluate, Assess—is not a "cookbook" method for students to apply mindlessly, but rather a tool for organizing students' thinking and discouraging equation hunting. It begins with an interpretation of the problem and an identification of the key

physics concepts involved; develops a plan for reaching the solution; carries out the mathematical evaluation; and assesses the solution to see that it makes sense, to compare the example with others, and to mine additional insights into physics. In nearly all of the text's worked examples, the Develop phase includes making a drawing, and most of these use a hand-drawn style to encourage students to make their own drawings—a step that research suggests they often skip. IDEA provides a common approach to all physics problem solving, an approach that emphasizes the conceptual unity of physics and helps break the typical student view of physics as a hodgepodge of equations and unrelated ideas. In addition to IDEA-based worked examples, other pedagogical features include:

- **Problem-Solving Strategy boxes** that follow the IDEA framework to provide detailed guidance for specific classes of physics problems, such as Newton's second law, conservation of energy, thermal-energy balance, Gauss's law, or multiloop circuits.
- **Tactics boxes** that reinforce specific essential skills such as differentiation, setting up integrals, vector products, drawing free-body diagrams, simplifying series and parallel circuits, or ray tracing.
- **QR codes** in the textbook allow students to link to video tutor demonstrations as they read, using their smartphones. These "Pause and predict" videos of key physics concepts ask students to submit a prediction before they see the outcome. The videos are also available in the Study Area of Mastering and in the Pearson eText.
- **GOT IT? boxes** that provide quick checks for students to test their conceptual understanding. Many of these use a multiple-choice or quantitative ranking format to probe student misconceptions and facilitate their use with classroom-response systems. Many new GOT IT? boxes have been added in the third edition, and now nearly every section of every chapter has at least one GOT IT? box.
- **Tips** that provide helpful problem-solving hints or warn against common pitfalls and misconceptions.
- **Chapter openers** that include a graphical indication of where the chapter lies in sequence as well as three columns of points that help make connections with other material throughout the book. These include a backward-looking "What You Know," "What You're Learning" for the present chapter, and a forward-looking "How You'll Use It." Each chapter also includes an opening photo, captioned with a question whose answer should be evident after the student has completed the chapter.
- **Applications**, self-contained presentations typically shorter than half a page, provide interesting and contemporary instances of physics in the real world, such as bicycle stability; flywheel energy storage; laser vision correction; ultracapacitors; noise-cancelling headphones; wind energy; magnetic resonance imaging; smartphone gyroscopes; combined-cycle power generation; circuit models of the cell membrane; CD, DVD, and Blu-ray technologies; radiocarbon dating; and many, many more.
- **For Thought and Discussion** questions at the end of each chapter designed for peer learning or for self-study to enhance students' conceptual understanding of physics.
- **Annotated figures** that adopt the research-based approach of including simple "instructor's voice" commentary to help students read and interpret pictorial and graphical information.
- **End-of-chapter** problems that begin with simpler exercises keyed to individual chapter sections and ramp up to more challenging and often multistep problems that synthesize chapter material. Context-rich problems focusing on real-world situations are interspersed throughout each problem set.
- **Chapter summaries** that combine text, art, and equations to provide a synthesized overview of each chapter. Each summary is hierarchical, beginning with the chapter's "big ideas," then focusing on key concepts and equations, and ending with a list of "applications"—specific instances or applications of the physics presented in the chapter.

Organization

This contemporary book is *concise*, *strategic*, and *progressive*, but it's *traditional* in its organization. Following the introductory Chapter 1, the book is divided into six parts. Part One (Chapters 2–12) develops the basic concepts of mechanics, including Newton's laws and conservation principles as applied to single particles and multiparticle systems. Part Two (Chapters 13–15) extends mechanics to oscillations, waves, and fluids. Part Three (Chapters 16–19) covers thermodynamics. Part Four (Chapters 20–29) deals with electricity and magnetism. Part Five (Chapters 30–32) treats optics, first in the geometrical optics approximation and then including wave phenomena. Part Six (Chapters 33–39) introduces relativity and quantum physics. Each part begins with a brief description of its coverage, and ends with a conceptual summary and a challenge problem that synthesizes ideas from several chapters.

Essential University Physics is available in two paperback volumes, so students can purchase only what they need—making the low-cost aspect of this text even more attractive. Volume 1 includes Parts One, Two, and Three, mechanics through thermodynamics. Volume 2 contains Parts Four, Five, and Six, electricity and magnetism along with optics and modern physics.

Instructor Supplements

NOTE: For convenience, all of the following instructor supplements can be downloaded from the Instructor's Resource Area of MasteringPhysics® (www.masteringphysics.com) as well as from the Instructor's Resource Center on www.pearsonglobaleditions.com/Wolfson.

- The **Instructor's Solutions Manual** contains solutions to all end-of-chapter exercises and problems, written in the Interpret/Develop/Evaluate/Assess (IDEA) problem-solving framework. The solutions are provided in PDF and editable Microsoft® Word formats for Mac and PC, with equations in MathType.
- **MasteringPhysics®** (www.masteringphysics.com) is the most advanced physics homework and tutorial system available. This online homework and tutoring system guides students through the toughest topics in physics with self-paced tutorials that provide individualized coaching. These assignable, in-depth tutorials are designed to coach students with hints and feedback specific to their individual errors. Instructors can also assign end-of-chapter problems from every chapter, including multiple-choice questions, section-specific exercises, and general problems. Quantitative problems can be assigned with numerical answers and randomized values (with sig fig feedback) or solutions. This third edition includes nearly 400 new problems written by the author explictly for use with MasteringPhysics.
- **Learning Catalytics** is a "bring your own device" student engagement, assessment, and classroom intelligence system that is based on cutting-edge research, innovation, and implementation of interactive teaching and peer instruction. With Learning Catalytics pre-lecture questions, you can see what students do and don't understand and adjust lectures accordingly.
- **Pearson eText** is available through MasteringPhysics®. Users can search for words or phrases, create notes, highlight text, bookmark sections, click on definitions to key terms, and launch PhET simulations and video tutor demonstrations as they read. Professors also have the ability to annotate the text for their course and hide chapters not covered in their syllabi.
- The **Test Bank** contains more than 2000 multiple-choice, true-false, and conceptual questions in TestGen® and Microsoft Word® formats for Mac and PC users. More than half of the questions can be assigned with randomized numerical values.

Student Supplements

- **MasteringPhysics®** (www.masteringphysics.com) is the most advanced physics homework and tutorial system available. This online homework and tutoring system guides students through the most important topics in physics with self-paced tutorials that provide individualized coaching. These assignable, in-depth tutorials are designed to coach students with hints and feedback specific to their individual errors. Instructors can also assign end-of-chapter problems from every chapter including multiple-choice questions, section-specific exercises, and general problems. Quantitative problems can be assigned with numerical answers and randomized values (with sig fig feedback) or solutions.
- **Pearson eText** is available through MasteringPhysics®. Allowing students access to the text wherever they have access to the Internet, Pearson eText comprises the full text with additional interactive features. Users can search for words or phrases, create notes, highlight text, bookmark sections, click on definitions to key terms, and launch PhET simulations and video tutor demonstrations as they read.

Acknowledgments

A project of this magnitude isn't the work of its author alone. First and foremost among those I thank for their contributions are the now several thousand students I've taught in calculus-based introductory physics courses at Middlebury College. Over the years your questions have taught me how to convey physics ideas in many different ways appropriate to your diverse learning styles. You've helped identify the "sticking points" that challenge introductory physics students, and you've showed me ways to help you avoid and "unlearn" the misconceptions that many students bring to introductory physics.

Thanks also to the numerous instructors and students from around the world who have contributed valuable suggestions for improvement of this text. I've heard you, and you'll find many of your ideas implemented in this third edition of *Essential University Physics*. And special thanks to my Middlebury physics colleagues who have taught from this text and who contribute valuable advice and insights on a regular basis: Jeff Dunham, Anne Goodsell, Noah Graham, Steve Ratcliff, and Susan Watson.

Experienced physics instructors thoroughly reviewed every chapter of this book, and reviewers' comments resulted in substantive changes—and sometimes in major rewrites—to the first drafts of the manuscript. We list all these reviewers below. But first, special thanks are due to several individuals who made exceptional contributions to the quality and in some cases the very existence of this book. First is Professor Jay Pasachoff of Williams College, whose willingness more than three decades ago to take a chance on an inexperienced coauthor has made writing introductory physics a large part of my professional career. Dr. Adam Black, physics editor and Ph.D. physicist at Pearson, had the vision to see promise in a new introductory text that would respond to the rising chorus of complaints about massive, encyclopedic, and expensive physics texts. Brad Patterson, developmental editor for the first edition, brought his graduate-level knowledge of physics to a role that made him a real collaborator. Brad is responsible for many of the book's innovative features, and it was a pleasure to work with him. John Murdzek and Matt Walker continued with Brad's excellent tradition of developmental editing on this third edition. We've gone to great lengths to make this book as error-free as possible, and much of the credit for that happy situation goes to Sen-Ben Liao, who solved every new and revised homework problem and updated the solutions manual.

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Preface to the Student

Welcome to physics! Maybe you're taking introductory physics because you're majoring in a field of science or engineering that requires a semester or two of physics. Maybe you're premed, and you know that medical schools are increasingly interested in seeing calculus-based physics on your transcript. Perhaps you're really gung-ho and plan to major in physics. Or maybe you want to study physics further as a minor associated with related fields like math or chemistry or to complement a discipline like economics, environmental studies, or even music. Perhaps you had a great high-school physics course, and you're eager to continue. Maybe high-school physics was an academic disaster for you, and you're approaching this course with trepidation. Or perhaps this is your first experience with physics. Whatever your reason for taking introductory physics, welcome!

And whatever your reason, my goals for you are similar: I'd like to help you develop an understanding and appreciation of the physical universe at a deep and fundamental level; I'd like you to become aware of the broad range of natural and technological phenomena that physics can explain; and I'd like to help you strengthen your analytic and quantitative problem-solving skills. Even if you're studying physics only because it's a requirement, I want to help you engage the subject and come away with an appreciation for this fundamental science and its wide applicability. One of my greatest joys as a physics teacher is having students tell me after the course that they had taken it only because it was required, but found they really enjoyed their exposure to the ideas of physics.

Physics is fundamental. To understand physics is to understand how the world works, both in everyday life and on scales of time and space so small and so large as to defy intuition. For that reason I hope you'll find physics fascinating. But you'll also find it challenging. Learning physics will challenge you with the need for precise thinking and language; with subtle interpretations of even commonplace phenomena; and with the need for skillful application of mathematics. But there's also a simplicity to physics, a simplicity that results because there are in physics only a very few really basic principles to learn. Those succinct principles encompass a universe of natural phenomena and technological applications.

I've been teaching introductory physics for decades, and this book distills everything my students have taught me about the many different ways to approach physics; about the subtle misconceptions students often bring to physics; about the ideas and types of problems that present the greatest challenges; and about ways to make physics engaging, exciting, and relevant to your life and interests.

I have some specific advice for you that grows out of my long experience teaching introductory physics. Keeping this advice in mind will make physics easier (but not necessarily easy!), more interesting, and, I hope, more fun:

- *Read* each chapter thoroughly and carefully before you attempt to work any problem assignments. I've written this text with an informal, conversational style to make it engaging. It's not a reference work to be left alone until you need some specific piece of information; rather, it's an unfolding "story" of physics—its big ideas and their applications in quantitative problem solving. You may think physics is hard because it's mathematical, but in my long experience I've found that failure to *read* thoroughly is the biggest single reason for difficulties in introductory physics.
- *Look for the big ideas*. Physics isn't a hodgepodge of different phenomena, laws, and equations to memorize. Rather, it's a few big ideas from which flow myriad applications, examples, and special cases. In particular, don't think of physics as a jumble of equations that you choose among when solving a problem. Rather, identify those few big ideas and the equations that represent them, and try to see how seemingly distinct examples and special cases relate to the big ideas.
- *When working problems*, *re-read* the appropriate sections of the text, paying particular attention to the worked examples. Follow the IDEA strategy described in Chapter 1 and used in every subsequent worked example. Don't skimp on the final Assess step. Always ask: Does this answer make sense? How can I understand my answer in relation to the big principles of physics? How was this problem like others I've worked, or like examples in the text?
- *Don't confuse physics with math*. Mathematics is a tool, not an end in itself. Equations in physics aren't abstract math, but statements about the physical world. Be sure you understand each equation for what it says about physics, not just as an equality between mathematical terms.
- *Work with others*. Getting together informally in a room with a blackboard is a great way to explore physics, to clarify your ideas and help others clarify theirs, and to learn from your peers. I urge you to discuss physics problems together with your classmates, to contemplate together the "For Thought and Discussion" questions at the end of each chapter, and to engage one another in lively dialog as you grow your understanding of physics, the fundamental science.

Detailed Contents

Volume 1 contains Chapters 1–19
Volume 2 contains Chapters 20–39

PART FOUR

Electromagnetism 434

Chapter 20 Electric Charge, Force, and Field 435
- 20.1 Electric Charge 435
- 20.2 Coulomb's Law 436
- 20.3 The Electric Field 439
- 20.4 Fields of Charge Distributions 442
- 20.5 Matter in Electric Fields 446

Chapter 21 Gauss's Law 455
- 21.1 Electric Field Lines 455
- 21.2 Electric Field and Electric Flux 457
- 21.3 Gauss's Law 460
- 21.4 Using Gauss's Law 462
- 21.5 Fields of Arbitrary Charge Distributions 468
- 21.6 Gauss's Law and Conductors 470

Chapter 22 Electric Potential 479
- 22.1 Electric Potential Difference 480
- 22.2 Calculating Potential Difference 483
- 22.3 Potential Difference and the Electric Field 488
- 22.4 Charged Conductors 490

Chapter 23 Electrostatic Energy and Capacitors 498
- 23.1 Electrostatic Energy 498
- 23.2 Capacitors 499
- 23.3 Using Capacitors 501
- 23.4 Energy in the Electric Field 505

Chapter 24 Electric Current 512
- 24.1 Electric Current 513
- 24.2 Conduction Mechanisms 515
- 24.3 Resistance and Ohm's Law 519
- 24.4 Electric Power 521
- 24.5 Electrical Safety 522

Chapter 25 Electric Circuits 529
- 25.1 Circuits, Symbols, and Electromotive Force 529
- 25.2 Series and Parallel Resistors 530
- 25.3 Kirchhoff's Laws and Multiloop Circuits 536
- 25.4 Electrical Measurements 538
- 25.5 Capacitors in Circuits 539

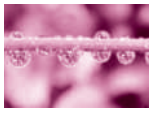
Chapter 26 Magnetism: Force and Field 549
- 26.1 What Is Magnetism? 550
- 26.2 Magnetic Force and Field 550
- 26.3 Charged Particles in Magnetic Fields 552
- 26.4 The Magnetic Force on a Current 555
- 26.5 Origin of the Magnetic Field 556
- 26.6 Magnetic Dipoles 559
- 26.7 Magnetic Matter 563
- 26.8 Ampère's Law 564

Chapter 27 Electromagnetic Induction 577
- 27.1 Induced Currents 578
- 27.2 Faraday's Law 579
- 27.3 Induction and Energy 583
- 27.4 Inductance 589
- 27.5 Magnetic Energy 594
- 27.6 Induced Electric Fields 597

Chapter 28 Alternating-Current Circuits 605
- 28.1 Alternating Current 605
- 28.2 Circuit Elements in AC Circuits 606
- 28.3 *LC* Circuits 610
- 28.4 Driven *RLC* Circuits and Resonance 613
- 28.5 Power in AC Circuits 616
- 28.6 Transformers and Power Supplies 617

Chapter 29 Maxwell's Equations and Electromagnetic Waves 623
29.1 The Four Laws of Electromagnetism 624
29.2 Ambiguity in Ampère's Law 624
29.3 Maxwell's Equations 626
29.4 Electromagnetic Waves 627
29.5 Properties of Electromagnetic Waves 631
29.6 The Electromagnetic Spectrum 634
29.7 Producing Electromagnetic Waves 635
29.8 Energy and Momentum in Electromagnetic Waves 636

PART FIVE



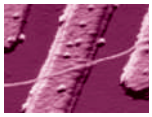
Optics 645

Chapter 30 Reflection and Refraction 646
30.1 Reflection 647
30.2 Refraction 648
30.3 Total Internal Reflection 651
30.4 Dispersion 652

Chapter 31 Images and Optical Instruments 659
31.1 Images with Mirrors 660
31.2 Images with Lenses 665
31.3 Refraction in Lenses: The Details 668
31.4 Optical Instruments 671

Chapter 32 Interference and Diffraction 679
32.1 Coherence and Interference 679
32.2 Double-Slit Interference 681
32.3 Multiple-Slit Interference and Diffraction Gratings 684
32.4 Interferometry 687
32.5 Huygens' Principle and Diffraction 690
32.6 The Diffraction Limit 693

PART SIX



Modern Physics 701

Chapter 33 Relativity 702
33.1 Speed *c* Relative to What? 703
33.2 Matter, Motion, and the Ether 703
33.3 Special Relativity 705
33.4 Space and Time in Relativity 706
33.5 Simultaneity Is Relative 712
33.6 The Lorentz Transformations 713
33.7 Energy and Momentum in Relativity 717
33.8 Electromagnetism and Relativity 720
33.9 General Relativity 721

Chapter 34 Particles and Waves 727
34.1 Toward Quantum Theory 728
34.2 Blackbody Radiation 728
34.3 Photons 730
34.4 Atomic Spectra and the Bohr Atom 734
34.5 Matter Waves 737
34.6 The Uncertainty Principle 739
34.7 Complementarity 741

Chapter 35 Quantum Mechanics 747
35.1 Particles, Waves, and Probability 748
35.2 The Schrödinger Equation 749
35.3 Particles and Potentials 751
35.4 Quantum Mechanics in Three Dimensions 758
35.5 Relativistic Quantum Mechanics 759

Chapter 36 Atomic Physics 764
36.1 The Hydrogen Atom 764
36.2 Electron Spin 768
36.3 The Exclusion Principle 771
36.4 Multielectron Atoms and the Periodic Table 772
36.5 Transitions and Atomic Spectra 776

Chapter 37 Molecules and Solids 782
37.1 Molecular Bonding 782
37.2 Molecular Energy Levels 784
37.3 Solids 787
37.4 Superconductivity 793

Chapter 38 Nuclear Physics 800
38.1 Elements, Isotopes, and Nuclear Structure 801
38.2 Radioactivity 806
38.3 Binding Energy and Nucleosynthesis 811
38.4 Nuclear Fission 813
38.5 Nuclear Fusion 819

Chapter 39 From Quarks to the Cosmos 827
39.1 Particles and Forces 828
39.2 Particles and More Particles 829
39.3 Quarks and the Standard Model 832
39.4 Unification 835
39.5 The Evolving Universe 837

APPENDICES

Appendix A Mathematics 847
Appendix B The International System of Units (SI) 855
Appendix C Conversion Factors 857
Appendix D The Elements 859
Appendix E Astrophysical Data 862

Answers to Odd-Numbered Problems 863
Credits 872
Index 875

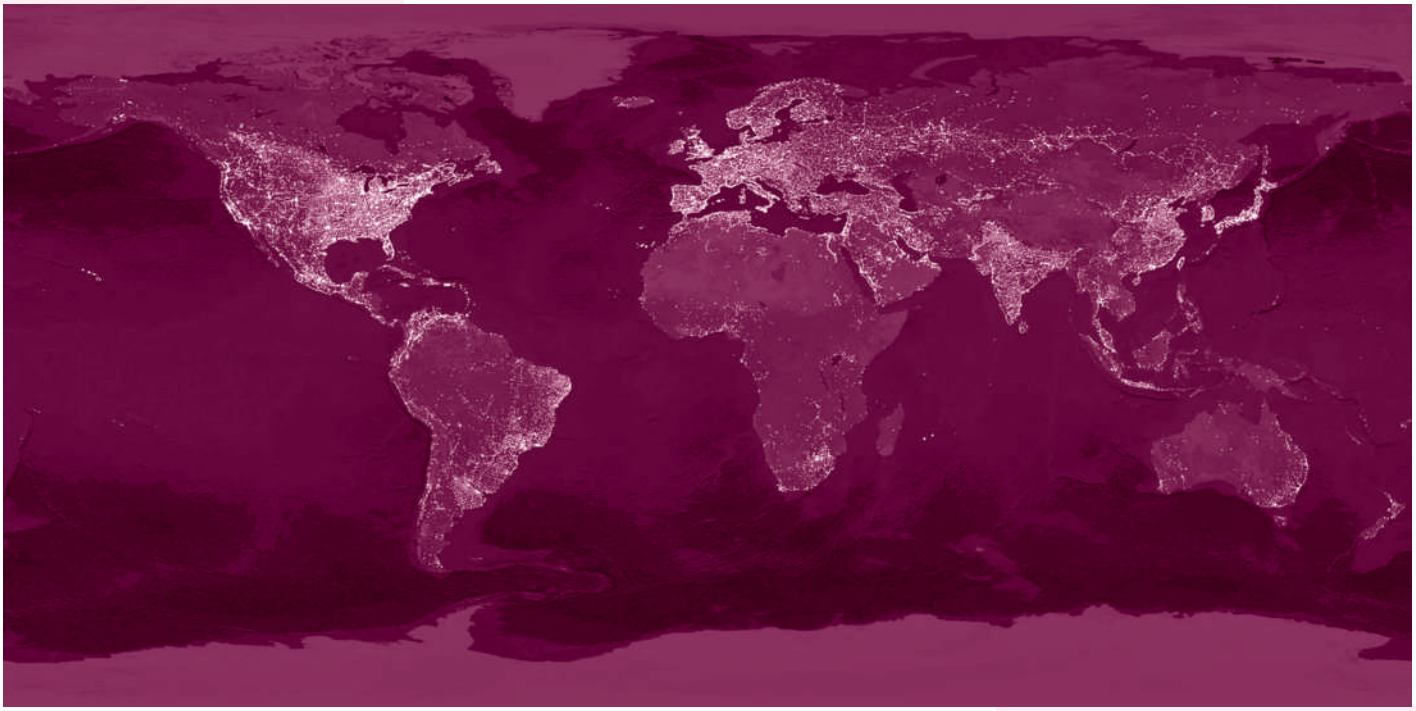
ESSENTIAL UNIVERSITY PHYSICS

VOLUME 2 CHAPTERS 20–39

PART FOUR

OVERVIEW

Electromagnetism



Electricity constitutes a significant portion of humankind's energy, as evidenced by this composite satellite image of Earth at night. Nearly all that electrical energy is produced by generators, devices that exploit an intimate relation between electricity and magnetism.

Electromagnetism is one of the fundamental forces, and it governs the behavior of matter from the atomic scale to the macroscopic world. Electromagnetic technology, from computer microchips to cell phones and on to large electric motors and generators, is essential to modern society. Even our bodies rely heavily on electromagnetism: Electric signals pace our heartbeat, electrochemical processes transmit nerve impulses, and the electric structure of cell membranes mediates the flow of materials into and out of the cell.

Four fundamental laws describe electricity and magnetism. Two deal separately with the two phenomena, while the others reveal profound connections that make electricity and magnetism aspects of a single phenomenon—electromagnetism. In this part you'll come to understand those fundamental laws, learn how electromagnetism determines the structure and behavior of nearly all matter, and explore the electromagnetic technologies that play so important a role in your life. Finally, you'll see how the laws of electromagnetism lead to electromagnetic waves and thus help us understand the nature of light.

Electric Charge, Force, and Field

What You Know

- You understand Newton's laws, so you know how to work with all kinds of forces.
- You know how to handle vector quantities.
- You're generally familiar with electricity and electronics on an everyday basis, and you probably know about electric charge and how like charges repel and opposite charges attract.

What You're Learning

- Electric charge is a fundamental property of matter, and here you'll learn how electric charge behaves and how two or more charges interact.
- You'll learn *Coulomb's law* for the *electric force* between two charges, written in vector form.
- You'll see how the *superposition principle* gives a simple prescription for combining electric forces from several charges.
- You'll review the *field concept* from Chapter 8 and learn how it applies to the *electric field*.
- You'll learn how to calculate the electric fields of distributions of multiple charges.
- You'll learn how matter behaves in electric fields and will learn the terms *conductor*, *insulator*, and *dielectric* as they apply to bulk matter.

How You'll Use It

- Electric charge and electric field are fundamental entities. You'll use these throughout your study of electromagnetism.
- A deeper understanding of electrical phenomena will also help you better appreciate everyday technology and use it more wisely and safely.

What's the fundamental criterion for initiating a lightning strike?

What holds your body together? What keeps a skyscraper standing? What holds your car on the road as you round a turn? What governs the electronic circuitry in your computer or smartphone, or provides the tension in your climbing rope? What enables a plant to make sugar from sunlight and simple chemicals? What underlies the awesome beauty of lightning? The answer, in all cases, is the **electric force**. With the exception of gravity, all the forces we've encountered in mechanics—including tension forces, normal forces, compression forces, and friction—are based on electric interactions; so are the forces responsible for all of chemistry and biology. The electric force, in turn, involves a fundamental property of matter—namely, electric charge.

20.1 Electric Charge

Electric charge is an intrinsic property of the electrons and protons that, along with uncharged neutrons, make up ordinary matter. What is electric charge? At the most fundamental level we don't know. We don't know what mass "really" is either, but we're familiar with it because we've spent our lives pushing objects around. Similarly, our knowledge of electric charge results from observing the behavior of charged objects.

Charge comes in two varieties, which Benjamin Franklin designated *positive* and *negative*. Those names are useful because the total charge on an object—the object's

net charge—is the algebraic sum of its constituent charges. Like charges repel, and opposites attract, a fact that constitutes a qualitative description of the electric force.

Quantities of Charge

All electrons carry the same charge, and all protons carry the same charge. The proton's charge has *exactly* the same magnitude as the electron's, but with opposite sign. Given that electrons and protons differ substantially in other properties—like mass—this electric relation is remarkable. Exercise 13 shows how dramatically different our world would be if there were even a slight difference between the magnitudes of the electron and proton charges.

The magnitude of the electron or proton charge is the **elementary charge** e. Electric charge is **quantized**; that is, it comes only in discrete amounts. In a famous experiment in 1909, the American physicist R. A. Millikan measured the charge on small oil drops and found it was always a multiple of a basic value we now know as the elementary charge.

Elementary particle theories show that the fundamental charge is actually $\frac{1}{3}e$. Such "fractional charges" reside on quarks, the building blocks of protons, neutrons, and many other particles. Quarks always join to produce particles with integer multiples of the full elementary charge, and it seems impossible to isolate individual quarks.

The SI unit of charge is the **coulomb** (C), named for the French physicist Charles Augustin de Coulomb (1736–1806). Since the late 19th century, the coulomb has been defined in terms of electric current. That will soon change, with the redefinition of SI units, to a definition based on assigning an exact value to the elementary charge e. Either way, it's convenient for our purposes to take $e \simeq 1.60\times10^{-19}$ C or to consider that 1 C is equivalent to about 6.24×10^{18} elementary charges.

Charge Conservation

Electric charge is a conserved quantity, meaning that the net charge in a closed region remains constant. Charged particles may be created or annihilated, but always in pairs of equal and opposite charge. The net charge always remains the same.

GOT IT? 20.1 The proton is a composite particle composed of three quarks, all of which are either *up quarks* (u; charge $+\frac{2}{3}e$) or *down quarks* (d; charge $-\frac{1}{3}e$). (More on quarks in Chapter 39.) Which of these quark combinations is the proton? (a) *udd*; (b) *uuu*; (c) *uud*; (d) *ddd*

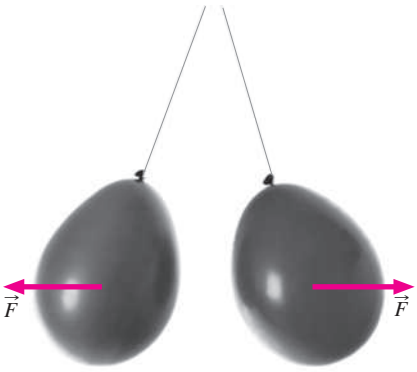


FIGURE 20.1 Two balloons carrying similar electric charges repel each other.

MP

PhET: Balloons and Static Electricity

20.2 Coulomb's Law

Rub a balloon; it gets charged and sticks to your clothing. Charge another balloon, and the two repel (Fig. 20.1). Socks cling to your clothes as they come from the dryer, and bits of Styrofoam cling annoyingly to your hands. Walk across a carpet, and you'll feel a shock when you touch a doorknob. All these are common examples where you're directly aware of electric charge.

Electricity would be unimportant if the only significant electric interactions were these obvious ones. In fact, the electric force dominates all interactions of everyday matter, from the motion of a car to the movement of a muscle. It's just that matter on a large scale is almost perfectly neutral, meaning it carries zero net charge. Therefore, electric effects aren't obvious. But at the molecular level, the electric nature of matter is immediately evident (Fig. 20.2).

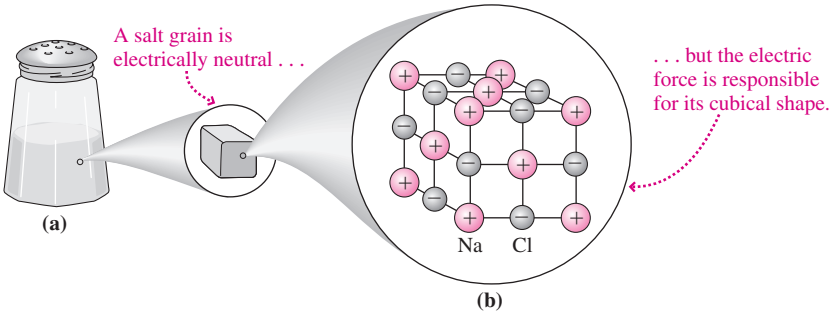


FIGURE 20.2 (a) A single salt grain is electrically neutral, so the electric force isn't obvious. (b) Actually, the electric force determines the structure of salt.

Attraction and repulsion of electric charges imply a force. Joseph Priestley and Charles Augustin de Coulomb investigated this force in the late 1700s. They found that the force between two charges acts along the line joining them, with the magnitude proportional to the product of the charges and inversely proportional to the square of the distance between them. **Coulomb's law** summarizes these results:

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r} \quad \text{(Coulomb's law)} \tag{20.1}$$

where $\vec{F}_{12}$ is the force charge q_1 exerts on q_2 and r is the distance between the charges. In SI the proportionality constant k has the approximate value $9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$. Force is a vector, and $\hat{r}$ is a unit vector that helps determine its direction. Figure 20.3 shows that $\hat{r}$ lies on a line passing through the two charges and points in the direction *from* q_1 *toward* q_2. Reverse the roles of q_1 and q_2, and you'll see that $\vec{F}_{21}$ has the same magnitude as $\vec{F}_{12}$ but the opposite direction; thus Coulomb's law obeys Newton's third law. Figure 20.3 also shows that the force is in the same direction as the unit vector when the charges have the same sign, but opposite the unit vector when the charges have different signs. Thus Coulomb's law accounts for the fact that like charges repel and opposites attract.

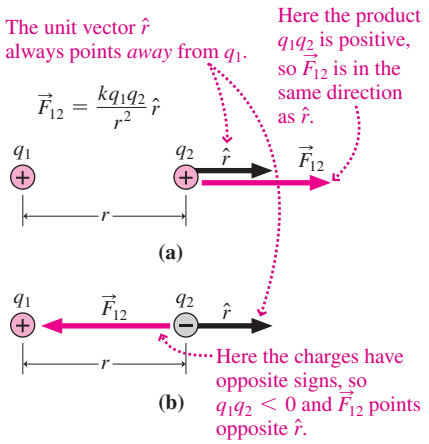


FIGURE 20.3 Quantities in Coulomb's law for calculating the force $\vec{F}_{12}$ that q_1 exerts on q_2.

PROBLEM-SOLVING STRATEGY 20.1 Coulomb's Law

The key to using Coulomb's law is to remember that force is a vector, and to realize that Coulomb's law in the form of Equation 20.1 gives both the magnitude and direction of the electric force. Dealing carefully with vector directions is especially important in situations with more than two charges.

INTERPRET First, make sure you're dealing with the electric force alone. Identify the charge or charges on which you want to calculate the force. Next, identify the charge or charges producing the force. These comprise the **source charge**.

DEVELOP Begin with a drawing that shows the charges, as in Fig. 20.4. If you're given charge coordinates, place the charges on the coordinate system; if not, choose a suitable coordinate system. For each source charge, determine the unit vector(s) in Equation 20.1. If the charges lie along or parallel to a coordinate axis, then the unit vector will be one of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, or $\hat{k}$, perhaps with a minus sign. In Fig. 20.4, the force on q_3 due to q_1 is such a case. When the two charges don't lie on a coordinate axis, like q_1 and q_2 in Fig. 20.4, you can find the unit vector by noting that the displacement vector $\vec{r}_{12}$ points in the desired direction, from the source charge to the charge experiencing the force. Dividing $\vec{r}_{12}$ by its own magnitude then gives the unit vector in the direction of $\vec{r}_{12}$; that is, $\hat{r} = \vec{r}_{12}/r_{12}$.

EVALUATE For each source charge, determine the electric force using Equation 20.1,

$$\vec{F}_{12} = (kq_1q_2/r^2)\hat{r}$$

with $\hat{r}$ the unit vector you've just found.

ASSESS As always, assess your answer to see that it makes sense. Is the direction of the force you found consistent with the signs and placements of the charges giving rise to the force?

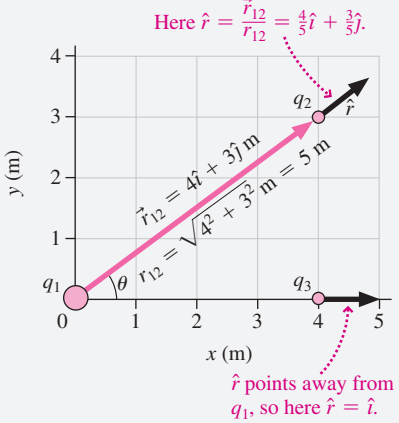


FIGURE 20.4 Finding unit vectors.

GOT IT? 20.2 Charge q_1 is located at $x = 1$ m, $y = 0$. What should you use for the unit vector $\hat{r}$ in Coulomb's law if you're calculating the force q_1 exerts on a charge q_2 located (1) at the origin and (2) at the point $x = 0, y = 1$ m? Explain why you can answer without knowing the sign of either charge.

EXAMPLE 20.1 Finding the Force: Two Charges

A 1.0-μC charge is at $x = 1.0$ cm, and a -1.5-μC charge is at $x = 3.0$ cm. What force does the positive charge exert on the negative one? How would the force change if the distance between the charges tripled?

INTERPRET Following our strategy, we identify the -1.5-μC charge as the one on which we want to find the force and the 1-μC charge as the source charge.

DEVELOP We're given the coordinates $x_1 = 1.0$ cm and $x_2 = 3.0$ cm. Our drawing, Fig. 20.5, shows both charges at their positions on the x-axis. With the source charge q_1 to the left, the unit vector in the direction from q_1 toward q_2 is $\hat{\imath}$.

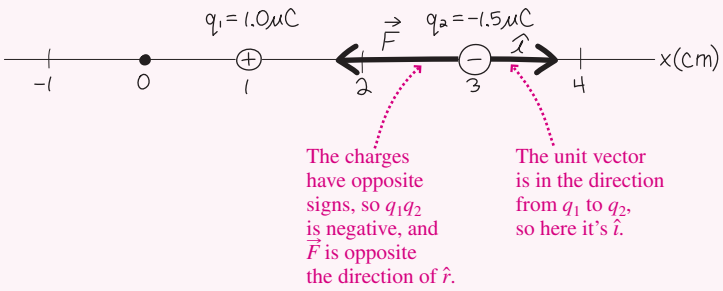


FIGURE 20.5 Sketch for Example 20.1.

EVALUATE Now we use Coulomb's law to evaluate the force:

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r}$$

$$= \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0\times10^{-6}\ \text{C})(-1.5\times10^{-6}\ \text{C})}{(0.020\ \text{m})^2}\hat{\imath}$$

$$= -34\hat{\imath}\ \text{N}$$

This force is for a separation of 2 cm; if that distance tripled, the force would drop by a factor of $1/3^2$, to $-3.8\hat{\imath}$ N.

ASSESS Make sense? Although the unit vector $\hat{\imath}$ points in the $+x$-direction, the charges have opposite signs and that makes the force direction opposite the unit vector, as shown in Fig. 20.5. In simpler terms, we've got two opposite charges, so they attract. That means the force exerted on a charge at $x = 3$ cm by an opposite charge at $x = 1$ cm had better be in the $-x$-direction. ■

CONCEPTUAL EXAMPLE 20.1 Gravity and the Electric Force

The electric force between elementary particles is far stronger than the gravitational force, yet gravity is much more obvious in everyday life. Why?

EVALUATE Gravity and the electric force obey similar inverse-square laws, and the magnitude of the force is proportional to the product of the masses or charges. There's a big difference, though: There's only one kind of mass, and gravity is always attractive, so large concentrations of mass—like a planet—result in strong gravitational forces. But charge comes in two varieties, and opposites attract, so large accumulations of matter tend to be electrically neutral, in which case large-scale electrical interactions aren't obvious.

ASSESS Ironically, it's the very strength of the electric force that makes it less obvious in everyday life. Opposite charges bind strongly, making bulk matter electrically neutral and its electrical interactions subtle.

MAKING THE CONNECTION Compare the magnitudes of the electric and gravitational forces between an electron and a proton.

EVALUATE Equation 8.1 gives the gravitational force: $F_g = Gm_em_p/r^2$. Equation 20.1 gives the electric force: $|F_E| = ke^2/r^2$, where we wrote e^2 because the electron and proton charges have the same magnitude. We aren't given the distance, but that doesn't matter because both forces have the same inverse-square dependence. The ratio of the force magnitudes is huge: $F_E/F_g = ke^2/Gm_em_p = 2.3\times10^{39}$!

Point Charges and the Superposition Principle

Coulomb's law is strictly true only for **point charges**—charged objects of negligible size. Electrons and protons can usually be treated as point charges; so, approximately, can any two charged objects if their separation is large compared with their size. But often we're interested in the electric effects of **charge distributions**—arrangements of charge spread over space. Charge distributions are present in molecules, memory cells in your computer,

your heart, and thunderclouds. We need to combine the effects of two or more charges to find the electric effects of such charge distributions.

Figure 20.6 shows two charges q_1 and q_2 that constitute a simple charge distribution. We want to know the net force these exert on a third charge q_3. To find that net force, you might calculate the forces $\vec{F}_{13}$ and $\vec{F}_{23}$ from Equation 20.1, and then vectorially add them. And you'd be right: The force that q_1 exerts on q_3 is unaffected by the presence of q_2, and vice versa, so you can apply Coulomb's law separately to the pairs q_1q_3 and q_2q_3 and then combine the results. That may seem obvious, but nature needn't have been so simple.

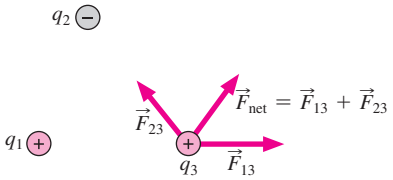


FIGURE 20.6 The superposition principle lets us add vectorially the forces from two or more charges.

The fact that electric forces add vectorially is called the **superposition principle**. Our confidence in this principle is ultimately based on experiments showing that electric and indeed electromagnetic phenomena behave according to the principle. With superposition we can solve relatively complicated problems by breaking them into simpler parts. If the superposition principle didn't hold, the mathematical description of electromagnetism would be far more complicated.

Although the force that one point charge exerts on another decreases with the inverse square of the distance between them, the same is not necessarily true of the force resulting from a charge distribution. The next example provides a case in point.

EXAMPLE 20.2 Finding the Force: Raindrops

Charged raindrops are ultimately responsible for lightning, producing substantial electric charge within specific regions of a thundercloud. Suppose two drops with equal charge q are on the x-axis at $x = \pm a$. Find the electric force on a third drop with charge Q at an arbitrary point on the y-axis.

INTERPRET Coulomb's law and the superposition principle apply, and we identify Q as the charge for which we want the force. The two charges q are the source charges.

DEVELOP Figure 20.7 is our drawing, showing the charges, the individual force vectors, and their sum. The drawing shows that the distance r in Coulomb's law is the hypotenuse $\sqrt{a^2 + y^2}$. It's clear from symmetry that the net force is in the y-direction, so we need to find only the y-components of the unit vectors. The y-components are clearly the same for each, and the drawing shows that they're given by $\hat{r}_y = y/\sqrt{a^2 + y^2}$.

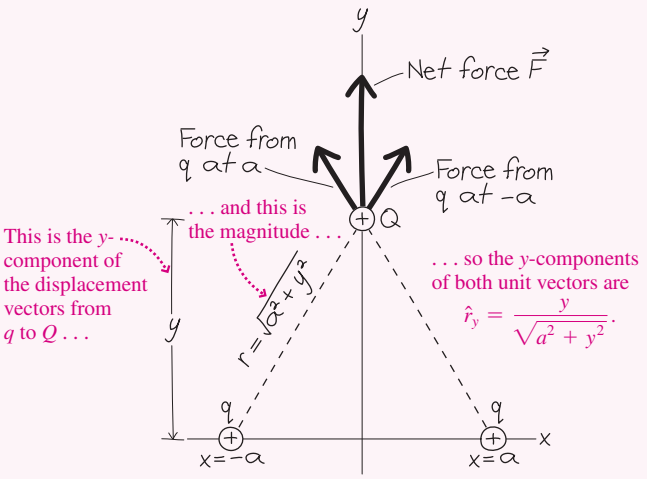


FIGURE 20.7 The force on Q is the vector sum of the forces from the individual charges.

EVALUATE From Coulomb's law, the y-component of the force from each q is $F_y = (kqQ/r^2)\hat{r}_y$, and the net force on Q becomes

$$\vec{F} = 2\left(\frac{kqQ}{a^2 + y^2}\right)\left(\frac{y}{\sqrt{a^2 + y^2}}\right)\hat{\jmath} = \frac{2kqQy}{(a^2 + y^2)^{3/2}}\hat{\jmath}$$

The factor of 2 comes from the two charges q, which contribute equally to the net force.

ASSESS Make sense? Evaluating $\vec{F}$ at $y = 0$ gives zero force. Here, midway between the two charges, Q experiences equal but opposite forces and the net force is zero. At large distances $y \gg a$, on the other hand, we can neglect a^2 compared with y^2, and the force becomes $\vec{F} = k(2q)Q\hat{\jmath}/y^2$. This is just what we would expect from a single charge $2q$ a distance y from Q—showing that the system of two charges acts like a single charge $2q$ at distances that are large compared with the charge separation. In between our two extremes the behavior of force with distance is more complicated; in fact, its magnitude initially increases as Q moves away from the origin and then begins to decrease.

In drawing Fig. 20.7, we tacitly assumed that q and Q have the same signs. But our analysis holds even if they don't; then the product qQ is negative, and the forces actually point opposite the directions shown in Fig. 20.7. ■

20.3 The Electric Field

In Chapter 8 we defined the gravitational field at a point as the gravitational force per unit mass that an object at that point would experience. In that context, we can think of $\vec{g}$ as the *force per unit mass* that any object would experience due to Earth's gravity. So we can picture the gravitational field as a set of vectors giving the magnitude and direction of the gravitational force per unit mass at each point, as shown in Fig. 20.8*a* on the next page.

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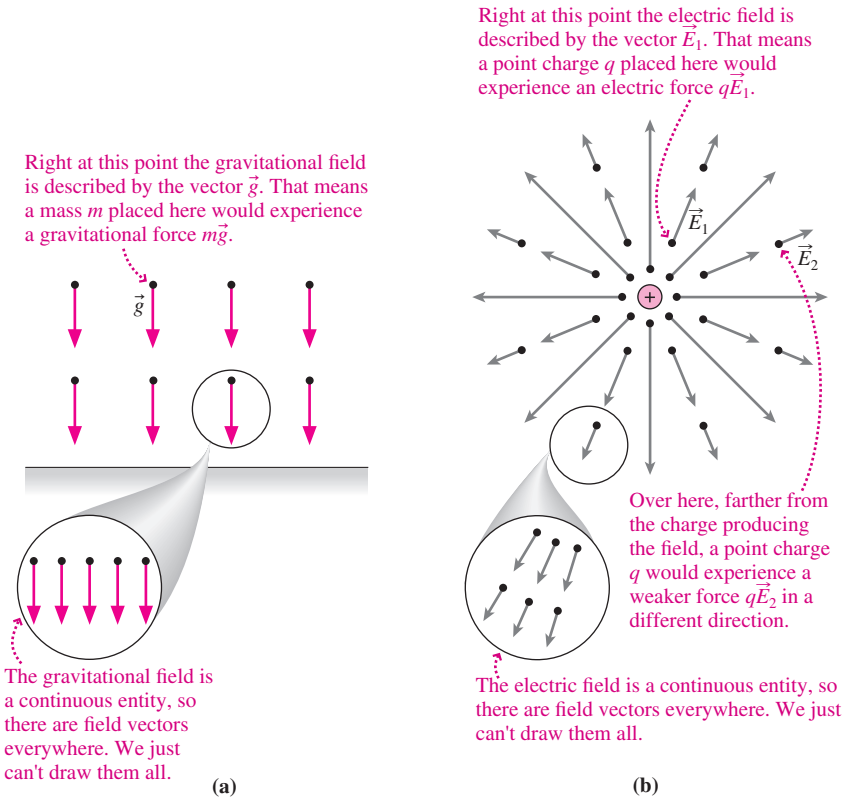


FIGURE 20.8 (a) Gravitational and (b) electric fields, here represented as sets of vectors.

We can do the same thing with the electric force, defining the **electric field** as the force per unit charge:

The electric field at any point is the force per unit charge that a charge would experience at that point. Mathematically,

$$\vec{E} = \frac{\vec{F}}{q} \qquad \text{(electric field)} \qquad (20.2a)$$

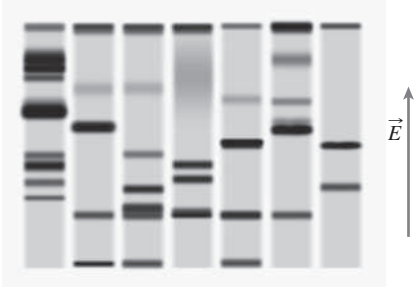
The electric field exists at every point in space. When we represent the field by vectors, we can't draw one everywhere, but that doesn't mean there isn't a field at all points. Furthermore, we draw vectors as extended arrows, but each vector represents the field at only one point—namely, the tail end of the vector. Figure 20.8*b* illustrates this for the electric field of a point charge.

The field concept leads to a shift in our thinking about forces. Instead of the action-at-a-distance idea that Earth reaches across empty space to pull on the Moon, the field concept says that Earth creates a gravitational field and the Moon responds to the field at its location. Similarly, a charge creates an electric field throughout the space surrounding it. A second charge then responds to the field at its immediate location. Although the field reveals itself only through its effect on a charge, the field nevertheless exists at all points, whether or not charges are present. Right now you probably find the field concept a bit abstract, but as you advance in your study of electromagnetism you'll come to appreciate that fields are an essential feature of our universe, every bit as real as matter itself.

We can use Equation 20.2a as a prescription for measuring electric fields. Place a point charge at some location, measure the electric force it experiences, and divide by the charge to get the field. In practice, we need to be careful because the field generally arises from some distribution of source charges. If the charge we're using to probe the field—the **test charge**—is large, the field it creates may disturb the source charges, altering their configuration and thus the field they create. For that reason, it's important to use a very small test charge.

APPLICATION **Electrophoresis**

Electrophoresis is a widely used application of electric fields for separating molecules by size and molecular weight. It's especially useful in biochemistry and molecular biology for distinguishing larger molecules like proteins and DNA fragments. In the commonly used *gel electrophoresis*, molecules carrying electric charge move through a semisolid but permeable gel under the influence of a uniform electric field; the greater the charge, the greater the electric force. The gel exerts a retarding force that increases with increasing molecular size, with the result that each molecular species moves at a velocity that depends on its size and charge. After a given time, the electric field is switched off. The locations of the molecules then serve as indicators of their size, with the molecules that traveled farthest being the smallest. The photo shows a typical gel electrophoresis result. Here DNA fragments were introduced into the seven channels at the top of the gel and then moved downward; their final locations indicate molecular size. The smaller molecules—those with fewer nucleotide base pairs—end up farther down on the gel. The electric field is shown by the arrow; it needs to point upward because DNA fragments carry a negative charge.



If we know the electric field $\vec{E}$ at a point, we can rearrange Equation 20.2a to find the force on any point charge q placed at that point:

$$\vec{F} = q\vec{E} \qquad \text{(electric force and field)} \tag{20.2b}$$

If the charge q is positive, then this force is in the same direction as the field, but if q is negative, then the force is opposite to the field direction.

Equations 20.2 show that the units of electric field are newtons per coulomb. Fields of hundreds to thousands of N/C are commonplace, while fields of 3 MN/C will tear electrons from air molecules.

EXAMPLE 20.3 Force and Field: Inside a Lightning Storm

A charged raindrop carrying 10 μC experiences an electric force of 0.30 N in the $+x$-direction. What's the electric field at its location? What would the force be on a -5.0-μC drop at the same point?

INTERPRET In this problem we need to distinguish between electric force and electric field. The electric field exists with or without the charged raindrop present, and the electric force arises when the charged raindrop is in the electric field.

DEVELOP Knowing the electric force and the charge on the raindrop, we can use Equation 20.2a, $\vec{E} = \vec{F}/q$, to get the electric field. Once we know the field, we can use Equation 20.2b, $\vec{F} = q\vec{E}$, to calculate the force that would act if a different charge were at the same point.

EVALUATE Equation 20.2a gives the electric field:

$$\vec{E} = \frac{\vec{F}}{q} = \frac{0.30\hat{\imath}\ \text{N}}{10\ \mu\text{C}} = 30\hat{\imath}\ \text{kN/C}$$

Acting on a -5.0-μC charge, this field would result in a force

$$\vec{F} = q\vec{E} = (-5.0\ \mu\text{C})(30\hat{\imath}\ \text{kN/C}) = -0.15\hat{\imath}\ \text{N}$$

ASSESS Make sense? The force on the second charge is opposite the direction of the field because now we've got a negative charge in the same field.

✓TIP The Field Is Independent of the Test Charge

Does the electric field in this example point in the $-x$-direction when the charge is negative? No. The field is independent of the particular charge experiencing that field. Here the electric field points in the $+x$-direction *no matter what charge* you put in the field. For a positive charge, the force $q\vec{E}$ points in the *same* direction as the field; for a negative charge, $q < 0$, the force is *opposite* the field.

■

The Field of a Point Charge

Once we know the field of a charge distribution, we can calculate its effect on other charges. The simplest charge distribution is a single point charge. Coulomb's law gives the force on a test charge q_{test} located a distance r from a point charge q: $\vec{F} = (kqq_{\text{test}}/r^2)\hat{r}$, where $\hat{r}$ is a unit vector pointing *away* from q. The electric field arising from q is the force per unit charge, or

$$\vec{E} = \frac{\vec{F}}{q_{\text{test}}} = \frac{kq}{r^2}\hat{r} \qquad \text{(field of a point charge)} \tag{20.3}$$

Since it's so closely related to Coulomb's law for the electric force, we also refer to Equation 20.3 as Coulomb's law. The equation contains no reference to the test charge q_{test} because the field of q exists independently of any other charge. Since $\hat{r}$ always points *away* from q, the direction of $\vec{E}$ is radially outward if q is positive and radially inward if q is negative. Figure 20.9 shows some field vectors for a negative point charge, analogous to those of the positive point charge in Fig. 20.8*b*.

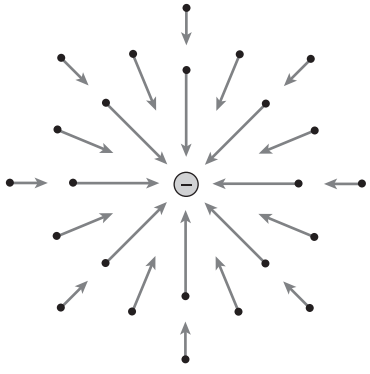


FIGURE 20.9 Field vectors for a negative point charge.

GOT IT? 20.3 A positive point charge is located at the origin of an x–y coordinate system, and an electron is placed at a location where the electric field due to the point charge is given by $\vec{E} = E_0(\hat{\imath} + \hat{\jmath})$, where E_0 is positive. Is the direction of the force on the electron (a) toward the origin, (b) away from the origin, (c) parallel to the x-axis, or (d) impossible to determine without knowing the coordinates of the electron's position?

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20.4 Fields of Charge Distributions

Since the electric force obeys the superposition principle, so does the electric field. That means the field of a charge distribution is the vector sum of the fields of the individual point charges making up the distribution:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots = \sum_i \vec{E}_i = \sum_i \frac{kq_i}{r_i^2}\hat{r}_i \tag{20.4}$$

Here the $\vec{E}_i$'s are the fields of the point charges q_i located at distances r_i from the point where we're evaluating the field—called, appropriately, the **field point**. The $\hat{r}_i$'s are unit vectors pointing *from* each point charge *toward* the field point. In principle, Equation 20.4 gives the electric field of *any* charge distribution. In practice, the process of summing the individual field vectors is often complicated unless the charge distribution contains relatively few charges arranged in a symmetric way.

Finding electric fields using Equation 20.4 involves the same strategy we introduced for finding the electric force; the only difference is that there's no charge to experience the force. The first step then involves identifying the field point. We still need to find the appropriate unit vectors and form the vector sum in Equation 20.4.

EXAMPLE 20.4 Finding the Field: Two Protons

Two protons are 3.6 nm apart. Find the electric field at a point between them, 1.2 nm from one of the protons. Then find the force on an electron at this point.

INTERPRET We follow our electric-force strategy, except that instead of identifying the charge experiencing the force, we identify the field point as being 1.2 nm from one proton. The source charges are the two protons; they produce the field we're interested in.

DEVELOP Let's have the protons define the x-axis, as drawn in Fig. 20.10. Then the unit vector $\hat{r}_1$ from the left-hand proton toward the field point (which we've marked P) is $+\hat{\imath}$, while $\hat{r}_2$ from the right-hand proton toward P is $-\hat{\imath}$.

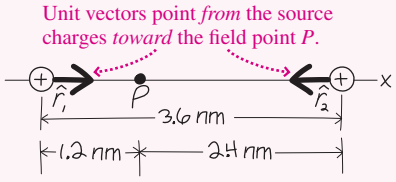

FIGURE 20.10 Finding the electric field at *P*.

EVALUATE We now evaluate the field at P using Equation 20.4:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{ke}{r_1^2}\hat{\imath} + \frac{ke}{r_2^2}(-\hat{\imath}) = ke\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)\hat{\imath}$$

We wrote e for q here because the protons' charge is the elementary charge.

Using $e = 1.6\times10^{-19}\,\text{C}$, $r_1 = 1.2\,\text{nm}$, and $r_2 = 2.4\,\text{nm}$ gives $\vec{E} = 750\hat{\imath}\,\text{MN/C}$. An electron at P will therefore experience a force $\vec{F} = qE = -eE = -0.12\hat{\imath}\,\text{nN}$.

ASSESS Make sense? The field points in the positive x-direction, reflecting the fact that P is closer to the left-hand proton with its stronger field at P. The force on the electron, on the other hand, is in the $-x$-direction; that's because the electron is negative (we used $q = -e$ for its charge), so the force it experiences is opposite the field. That field of almost 1 GN/C sounds huge—but that's not unusual at the microscopic scale, where we're close to individual elementary particles. ■

Sometimes we're interested in finding not the electric field but a point or points where the field has a particular value—often zero. Conceptual Example 20.2 explores such a case.

CONCEPTUAL EXAMPLE 20.2 Zero Field, Zero Force

A positive charge $+2Q$ is located at the origin, and a negative charge $-Q$ is at $x = a$. In which region of the x-axis is there a point where the force on a test charge—and therefore the electric field—is zero?

INTERPRET We're asked to locate qualitatively a point where the field is zero. Our sketch of the situation, Fig. 20.11, shows that the two charges divide the x-axis into three regions: (1) to the left of $2Q$ ($x < 0$), (2) between the charges ($0 < x < a$), and (3) to the right of $-Q$ ($x > a$). We need to determine which region could include a point where the electric force on a test charge is zero.

EVALUATE Consider what would happen to a positive test charge placed in each of these three regions. Anywhere in region (1), the test charge is closer to the charge with greater magnitude ($2Q$). That

charge dominates throughout region (1), where our test charge would experience a repulsive force (to the left). The electric field, then, can't be zero in region (1). Between the two charges, the repulsive force from $2Q$ on a positive test charge points to the right; so does the attractive force from $-Q$. The field, therefore, can't be zero in region (2). That leaves region (3). Could the field be zero here? Put a positive test charge very close to $-Q$, and it experiences an attractive force toward the left. But far away, the distance between $2Q$ and $-Q$ becomes negligible. The fields of both charges drop off as the inverse square of the distance, so at large distances the field of the stronger charge will dominate. Therefore there *is* a point somewhere to the right of $-Q$ where the force on a test charge, and therefore the electric field, will be zero.

ASSESS This answer is consistent with our insight from Example 20.2 that when we get far from a charge distribution it begins to resemble a point charge with the net charge of the distribution. Here that net charge is $2Q - Q = +Q$, so at large distances we should indeed have a field pointing away from the charge distribution—and that's to the right in region (3). Although we considered a positive test charge, you'll reach the same conclusion with a negative test charge.

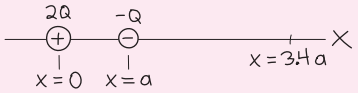

FIGURE 20.11 Where is the electric field zero? We've marked the answer, at $x = 3.4a$.

MAKING THE CONNECTION Find an expression for the position where the electric field in this example is zero.

EVALUATE In Fig. 20.11 we've taken the origin at $2Q$, so at any position x in region (3) we're a distance x from $2Q$ and a distance $x-a$ from $-Q$. Since we're to the right of both charges, the unit vector in Equation 20.3 for the point-charge field—a vector that always points away from the point charge—becomes $+\hat{\imath}$ for both charges. Applying Equation 20.3, $\vec{E} = (kq/r^2)\hat{r}$, for the fields of the two charges and summing gives

$$\vec{E} = \frac{k(2Q)}{x^2}\hat{\imath} + \frac{k(-Q)}{(x-a)^2}\hat{\imath}$$

If we set this expression to zero, we can cancel k, Q, and $\hat{\imath}$; inverting both sides of the remaining equation gives $x^2/2 = (x-a)^2$. Finally, taking the square root and solving for x gives the answer: $x = a\sqrt{2}/(\sqrt{2}-1) \simeq 3.4a$. As a check, note that this point does indeed lie to the right of $x = a$. We've marked this point in Fig. 20.11.

The Electric Dipole

One of the most important charge distributions is the **electric dipole**, consisting of two point charges of equal magnitude but opposite sign. Many molecules are essentially dipoles, so understanding the dipole helps explain molecular behavior (Fig. 20.12). During contraction the heart muscle becomes essentially a dipole, and physicians performing electrocardiography are measuring, among other things, the strength and orientation of that dipole. Technological devices, including radio and TV antennas, often use the dipole configuration.

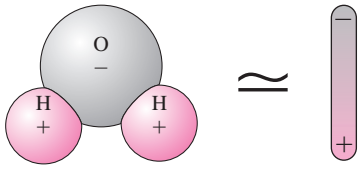

FIGURE 20.12 A water molecule behaves like an electric dipole. Its net charge is zero, but regions of positive and negative charge are separated.

EXAMPLE 20.5 The Electric Dipole: Modeling a Molecule

A molecule may be modeled approximately as a positive charge q at $x = a$ and a negative charge $-q$ at $x = -a$. Evaluate the electric field on the y-axis, and find an approximate expression valid at large distances ($|y| \gg a$).

INTERPRET Here's another example where we'll use our strategy in applying Equation 20.4 to calculate the field of a charge distribution. We identify the field point as being anywhere on the y-axis and the source charges as being $\pm q$.

DEVELOP Figure 20.13 is our drawing. The individual unit vectors point from the two charges toward the field point, but the *negative* charge contributes a field *opposite* its unit vector; we've indicated the individual fields in Fig. 20.13. Here symmetry makes the y-components cancel, giving a net field in the $-x$-direction. So we need only the x-components of the unit vectors, which Fig. 20.13 shows are $\hat{r}_{x-} = a/r$ for the negative charge at $-a$ and $\hat{r}_{x+} = -a/r$ for the positive charge at a.

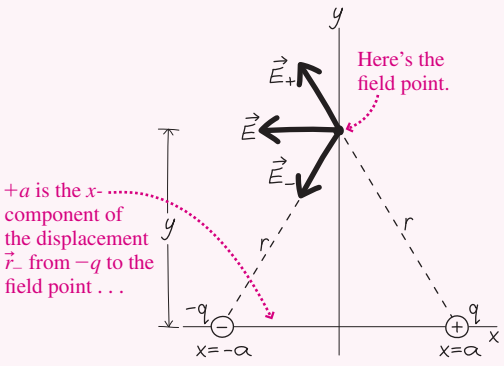

FIGURE 20.13 Finding the field of an electric dipole.

(*continued*)

EVALUATE We then evaluate the field using Equation 20.4:

$$\vec{E} = \frac{k(-q)}{r^2}\left(\frac{a}{r}\right)\hat{\imath} + \frac{kq}{r^2}\left(-\frac{a}{r}\right)\hat{\imath} = -\frac{2kqa}{(a^2 + y^2)^{3/2}}\hat{\imath}$$

where in the last step we used $r = \sqrt{a^2 + y^2}$. For $|y| \gg a$ we can neglect a^2 compared with y^2, giving

$$\vec{E} \simeq -\frac{2kqa}{|y|^3}\hat{\imath} \qquad (|y| \gg a)$$

ASSESS Make sense? The dipole has no net charge, so at large distances its field can't have the inverse-square drop-off of a point-charge field. Instead the dipole field falls faster, here as $1/|y|^3$. Note that we were careful to put absolute value signs on y^3; that way, our result applies for both positive and negative values of y.

✓TIP Approximations

Making approximations requires care. Here we're basically asking for the field when y is so large that a is negligible *compared with* y. So we neglect a^2 compared with y^2 when the two are summed, but we *don't* neglect a when it appears in the numerator, where it isn't being directly compared with y.

■

Example 20.5 shows that the dipole field at large distances decreases as the inverse *cube* of distance. Physically, that's because the dipole has zero *net* charge. Its field arises entirely from the slight separation of two opposite charges. Because of this separation the dipole field isn't exactly zero, but it's weaker and more localized than the field of a point charge. Many complicated charge distributions exhibit the essential characteristic of a dipole—that is, they're neutral but consist of separated regions of positive and negative charge—and at large distances such distributions all have essentially the same field configuration.

At large distances the dipole's physical characteristics q and a enter the equation for the electric field only through the product qa. We could double q and halve a, and the dipole's electric field would remain unchanged. At large distances, therefore, a dipole's electric properties are characterized completely by its **electric dipole moment** p, defined as the product of the charge q and the separation d between the two charges making up the dipole:

$$p = qd \quad \text{(dipole moment)} \tag{20.5}$$

In Example 20.5 the charge separation was $d = 2a$, so there the dipole moment was $p = 2aq$. In terms of the dipole moment, the field in Example 20.5 can then be written

$$\vec{E} = -\frac{kp}{|y|^3}\hat{\imath} \qquad \begin{pmatrix}\text{dipole field for } |y| \gg a,\\ \text{on perpendicular bisector}\end{pmatrix} \tag{20.6a}$$

You can show in Problem 52 that the field on the dipole axis is given by

$$\vec{E} = \frac{2kp}{|x|^3}\hat{\imath} \qquad \begin{pmatrix}\text{dipole field}\\ \text{for } |x| \gg a, \text{ on axis}\end{pmatrix} \tag{20.6b}$$

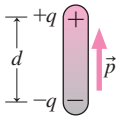

FIGURE 20.14 The dipole moment vector has magnitude $p = qd$ and points from the negative toward the positive charge.

Because the dipole isn't spherically symmetric, its field depends not only on distance but also on orientation; for instance, Equations 20.6 show that the field along the dipole axis at a given distance is twice as strong as along the bisector. So it's important to know the orientation of a dipole in space, and therefore we generalize our definition of the dipole moment to make it a vector of magnitude $p = qd$ in the direction from the negative toward the positive charge (Fig. 20.14).

GOT IT? 20.4 Far from a charge distribution, you measure an electric field strength of 800 N/C. What will the field strength be if you double your distance from the charge distribution, if the distribution consists of (1) a point charge or (2) a dipole?

Continuous Charge Distributions

Although any charge distribution ultimately consists of pointlike electrons and protons, it would be impossible to sum all the field vectors from the 10^{23} or so particles in a typical piece of matter. Instead, it's convenient to make the approximation that charge is spread continuously over the distribution. If the charge distribution extends throughout a volume, we describe it in terms of the **volume charge density** ρ, with units of C/m^3. For charge distributions spread over surfaces or lines, the corresponding quantities are the **surface charge density** σ (C/m^2) and the **line charge density** λ (C/m).

To calculate the field of a continuous charge distribution, we divide the charged region into very many small charge elements dq, each small enough that it's essentially a point charge. Each dq then produces an electric field $d\vec{E}$ given by Equation 20.3: $d\vec{E} = (k\,dq/r^2)\hat{r}$. We then form the vector sum of all the $d\vec{E}$'s (Fig. 20.15). In the limit of infinitely many infinitesimally small dq's and their corresponding $d\vec{E}$'s, that sum becomes an integral and we have

$$\vec{E} = \int d\vec{E} = \int \frac{k\,dq}{r^2}\hat{r} \qquad \begin{pmatrix}\text{field of a continuous}\\ \text{charge distribution}\end{pmatrix} \tag{20.7}$$

The limits of this integral include the entire charge distribution.

Calculating the field of a continuous charge distribution involves the same strategy we've already used: We identify the field point and the source charges—although now the source is a continuous charge distribution. Summing the individual field contributions now presents us with an integral, and that means writing the unit vectors $\hat{r}$ and distances r in terms of coordinates over which we can integrate. Setting up the integral involves the same strategy we outlined in Chapter 9 to find the center of mass of a continuous distribution of matter, and used again in Chapter 10 to find rotational inertias.

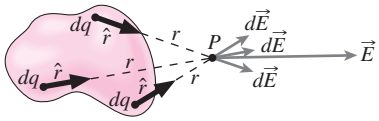

FIGURE 20.15 The electric field at P is the vector sum of the fields $d\vec{E}$ arising from the individual charge elements dq, each calculated using the appropriate distance r and unit vector $\hat{r}$.

EXAMPLE 20.6 Evaluating the Field: A Charged Ring

A ring of radius a carries a charge Q distributed evenly over the ring. Find an expression for the electric field at any point on the axis of the ring.

INTERPRET We identify the field point as lying anywhere on the ring's axis, and the source charge as the entire ring.

DEVELOP Let's take the x-axis to coincide with the ring axis, with the center of the ring at $x = 0$ (Fig. 20.16). The figure shows that the y-components of the field contributions from pairs of charge elements on opposite sides of the ring cancel; therefore, the net field points in the $+x$-direction (for $x > 0$) and we need only the x-components of the unit vectors. Those are the same for all unit vectors—namely, $\hat{r}_x = x/r$.

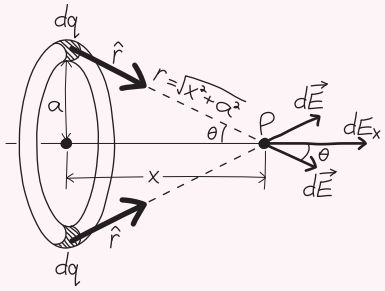

FIGURE 20.16 The electric field of a charged ring points along the ring axis, since field components perpendicular to the axis cancel in pairs.

EVALUATE We're now ready to set up the integral in Equation 20.7. Here each charge element contributes the same amount $dE_x = (k\,dq/r^2)\hat{r}_x = (k\,dq/r^2)(x/r)$ to the field. Figure 20.16 shows that $r = \sqrt{x^2 + a^2} = (x^2 + a^2)^{1/2}$, so the integral becomes

$$E = \int_{\text{ring}} dE_x = \int_{\text{ring}} \frac{kx\,dq}{(x^2 + a^2)^{3/2}} = \frac{kx}{(x^2 + a^2)^{3/2}} \int_{\text{ring}} dq$$

The last step follows because we have a fixed field point P, so its coordinate x is a constant for the integration. But the remaining integral is just the sum of all the charge elements on the ring—namely, the total charge Q. So our result becomes

$$E = \frac{kQx}{(x^2 + a^2)^{3/2}} \qquad \text{(on-axis field, charged ring)}$$

This is the magnitude; the direction is along the x-axis, away from the ring if Q is positive and toward it if Q is negative.

ASSESS Make sense? At $x = 0$ the field is zero. A charge placed at the ring center is pulled (or pushed) equally in all directions—no net force, so no electric field. But for $x \gg a$, we get $E = kQ/x^2$—just what we expect for a point charge Q. As always, a finite-size charge distribution looks like a point charge at large distances. ■